STA6557 Object Data Analysis SPRING 2022

Days/Time/Room: TT 11:15 AM -12:30 AM, OSB108 FLEX

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Textbook: *Nonparametric Statistic on Manifolds and their Applications to Object Data Analysis, by Vic Patrangenaru, Leif Ellingson*, 2015, Chapman Hall/CRC, ISBN-13: 978-1439820506, ISBN-10: 1439820503 .

Prerequisite: STA 5707 or equivalent

Course outcomes: After completing this course, the student will be able to:

- determine what is the appropriate object space for analyzing a given collection of observations of digital images, medical imaging outputs, shapes, unit vectors, functions, phylogenetic trees, etc.
- apply key results in modern statistics, including the central limit theorems or other asymptotic theorems for nonparametric inference on manifolds
- use nonparametric statistical procedures for statistical inference, including large sample theory and nonparametric bootstrap on an object space with a manifold stratification structure
- analyze object data

Course description: Object data analysis is the most general type of data analysis that is known in Statistics. Technically, this is Data analysis on object spaces with a mani-

fold stratification. Data on manifolds originally arose in astronomy, meteorology, geology, cartography, biology and physics. A drastic increase in the need of data analysis on manifolds occurred with the computer revolution in digital imagery, and the internet. Digital images, which arguably account for the largest types of data available, are today the bread and butter of modern sciences such as Digital Image Analysis, Bioinformatics, Medical Imaging, Computer Vision, Pattern Recognition, Astrophysics, Learning, Earth Sciences, Forensics, etc. Since manifolds are most often nonlinear spaces, fundamental statistical indices of a probability distribution on a manifold, such as mean and covariance have to be redefined in this general context, and their estimation is naturally approached from a nonparametric perspective. The foundation of Nonparametric Statistics on manifolds includes large sample theory on manifolds, density estimation on manifolds, inference on manifolds, and other techniques originating from multivariate analysis. After a recollection of nonparametric multivariate analysis, and a rigorous introduction to abstract manifolds and manifolds as sample spaces, one defines location and spread parameters for distribution on manifolds, and proves the consistency of their sample counterparts. Hypothesis testing and density estimation on manifolds follow. The second part is dedicated to a nonparametric statistical analysis on certain special manifolds arising in statistics, such as axial manifolds, various shape manifolds, Diffusion Tensor Imaging manifolds, learning manifolds, etc. The third part focuses on concrete applications in Genetics, Astronomy, Image Analysis, Medical Imaging, Bioinformatics, Computer Vision and 3D scene recognition, etc.

Topics covered

1. Data on Manifolds and on Stratified Space

1.1 Directional and Axial Data.

1.2 Similarity Shape Data and Size and Shape Data

1.3 Digital Camera Imaging Data

1.4 CT scan data

1.5 MRI-DTI data

2. Review of Nonparametric Multivariate Inference.

2.1 Basic Probability Theory.

2.2 Integration on Euclidean Spaces

2.3 Random vectors.

2.4 Parameters and Sampling Distributions of their Estimators

2.5 Consistency and Asymptotic Distributions of Estimators.

2.6 The Multivariate Normal Distribution

2.7 Convergence in Distribution and The Central Limit Theorem.

2.8 Basic Large Sample Theory

2.9 Elementary Nonparametric Inference

2.10 Principal Components Analysis (P.C.A.)

2.11. Multidimensional Scaling

2.12 Nonparametric Bootstrap and the Edgeworth Expansion.

2.13 Nonparametric Function Estimation

3. Basic Geometry of Manifolds

3.1 Manifolds. Submanifolds, Embeddings, Lie Group actions, Hilbert manifolds

3.2 Riemannian Structures. Curvature. Geodesics.

3.3 Topology of Manifolds

3.3.Basic Algebraic and Differential Topology

3.4. Manifolds in Statistics

4. One sample Inference on Manifolds.

4.1. Fréchet means.

4.2. Consistency of Fréchet sample means

5.2 Asymptotic and bootstrap distributions of Fréchet total variances on manifolds.

5.4 A central limit theorem for Fréchet sample means and bootstrapping.

5.5 The CLT for extrinsic sample means and confidence regions for the extrinsic mean.

5. Two sample tests on Manifolds.

6.1 Introduction

6.2 A two sample test for total extrinsic variances.

6.3 Test for mean change in matched pairs.

6.4 A large sample test for comparing extrinsic means of two independent populations on a manifold.

6.5 The two sample problem on a Riemannian homogeneous space.

6.6 Nonparametric Bootstrap for Two Sample Tests.

7. Function estimation on Manifolds

7.1 Function estimation on Homogeneous spaces and Lie groups

7.2 Statistical Inverse Estimation

Applications of Nonparametric Statistics on Manifolds

15.1 Diffusion Tensor Imaging Brain Analysis.

15.2. Tests for Equality of Generalized Frobenius Means via Cholesky Decompositions

16. Applications to Directional Data Analysis.

16.1 The Pluto controversy

16.2 The solar nebula theory

16.3 Why Pluto is not a major planet of the Solar System.

17 Applications of Direct Similarity Shape Analysis.

17.1. University School X-ray data analysis

17.3 Louisiana Experimental Glaucoma Study data analysis

18 Similarity Shape Analysis.

18.2 Planar direct similarity shape manifolds

18.3 Reflection shape manifolds in higher dimensions.

18.5 Extrinsic mean planar shapes and and their estimators

18.6 Asymptotic distribution of mean shapes

19. Direct Similarity Shape Analysis of Planar Contours.

20. Estimation of Mean Skull Size and Shape from CT scans, with Applications in Planning Reconstructive Plastic Surgery in Young Adults.

21. A statistical Analysis of 2D Electrophoretic Gels based on Affine Shapes 358 23. Applications to 3D machine vision. 23.1. Projective Shape Analysis of 3D Scenes from Pinhole Camera Images.

23.3. Projective Shape and 3D Reconstruction

23.4. Nonparametric Estimation and Testing for the Projective Shape of a 3D Configuration

23.5. Face recognition example.

24. Two Sample Tests for Mean Projective Shapes and Scene Recognition

25. Mean Glaucomatous Projective Shape Change Detection From Stereo Pair Images.

26. Computational Issues.

27. Statistics on Stratified Spaces. Sticky means.

Grading: The course grade will be calculated on the basis of a midterm exam, (50%), and a final presentation(50%). The final project presentation is meant to be created as a draft of a paper to be submitted on ArXiv (or equivalent).

University Attendance Policy: Excused absences include documented illness, deaths in the family and other documented crises, call to active military duty or jury duty, religious holy days, and official University activities. These absences will be accommodated in a way that does not arbitrarily penalize students who have a valid excuse. Consideration will also be given to students whose dependent children experience serious illness.

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Americans With Disabilities Act: Students with disabilities needing academic accommodation should: (1) register with and provide documentation to the Student Disability Resource Center; and (2) bring a letter to the instructor indicating the need

for accommodation and what type. This should be done during the first week of class.

This syllabus and other class materials are available in alternative format upon request.

For more information about services available to FSU students with disabilities, contact the: Student Disability Resource Center

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http://www.disabilitycenter.fsu.edu/

Disclaimer: This syllabus provides a general plan; deviations may be necessary.