STA 5334 Limit Theory of Statistics, Spring 2023

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Days/Time/OSB 110: TT 11:35 AM - 12:50 PM
Office hours: Th 10:15 AM - 11:15 AM
Texts:
-A Course in large Sample Theory, by **Thomas Ferguson**. Publisher: CRC Press 1996
-A Course in Mathematical Statistics and Large Sample Theory, by **Rabi Bhattacharya, Lizhen Lin and Vic Patrangenaru**, Springer, Statistics Series. New York, USA, 2016. ISBN 978-1-4939-4032-5
Additionsl topics may include chapters from a. the first part of: Nonparametric Statistics on Manifolds and their Applications to Object Data Analysis,, by **Vic Patrangenaru and Leif Ellingson**. ISBN-13: 9781439820506, ISBN-10: 1439820503 Publisher: CRC Press 2015
and from
b. Topological Data Analysis
Prerequisite: STA 5327 or STA 5707.
Course description: This course provides the **most important knowledge a future mathematical statistician should master**. The course provides definitions, **full detailed proofs** of critical fundamental results in everyday's Statistics, including, but not limited to convergence in distribution of random vectors, laws of large numbers, central limit theorems and asymptotic distributions, asymptotic efficiency, rates of convergence, nonparametric confidence regions, density estimation, asymptotic statistics on metric spaces, including for sample quantiles and data depth estimation. The first part of the course we be concerned with foundational material and limit theory chapters from the recommended texts:

- Modes of convergence in Probability Theory and CLT (Sections 1-5- from Ferguson)
- Ch. 6. Consistency and Asymptotic Distributions of Statistics. *Example: Generalized Slutsky theorem (multiparameter case), asymptotic distributions of T^2-like statistics*
- Ch. 7. Large Sample of Estimation in Parametric Models *Example: Asymptotic efficiency of the m.l.e.-multiparameter case*
- Ch. 8. Tests in Parametric and Nonparametric Models. *Example: generalized Wilks lambda. One calculates the likelihood ratio test statistic*

$$\Lambda_n = \frac{\max_{\boldsymbol{\theta}\in\Theta_0} \tilde{f}_n(\mathbf{X};\boldsymbol{\theta})}{\max_{\boldsymbol{\theta}\in\Theta} \tilde{f}_n(\mathbf{X};\boldsymbol{\theta})} = \frac{\tilde{f}_n(\mathbf{X};\widehat{\widehat{\boldsymbol{\theta}}}_n)}{\tilde{f}_n(\mathbf{X};\widehat{\boldsymbol{\theta}}_n)} \,. \tag{1}$$

The test statistic is then

$$\lambda_n = -2\log\Lambda_n = 2\log\tilde{f}_n(\mathbf{X};\widehat{\boldsymbol{\theta}}_n) - 2\log\tilde{f}_n(\mathbf{X};\widehat{\widehat{\boldsymbol{\theta}}}_n) \tag{2}$$

and under the null hypothesis $\boldsymbol{\theta} \in \Theta_0$, λ_n *converges in law to a r.v. having a chi-square distribution with* k *d.f., where* k *is the codimension of* Θ_0 *in* Θ.

- Ch. 9. The Nonparametric Bootstrap *Example: improved coverage error of nonparametric bootstrap estimation vs asymptotic estimation*

For the second part, students are assume to work on limit theory related final exam projects, that will help them get a strong grip on Statistics at the knowledge level of a tenure track faculty in Mathematical and Statistical Sciences, helping them advance towards their Statistics, Biostatistics or Data Science degree.

Attendance policy: Active attendance adds up to 5 bonus points. On the other hand, if you miss at least 3 times in a row, this extracredit is lost.
Grading: The course grade will be calculated on the basis of one midterm in class exam and a final project (50% each). The test will be announced in class.
Assigned homework will be discussed in class or/and office hours.
Disclaimer: This syllabus provides a general plan; deviations may be necessary.