

Homework #6  
STA 4442/5440: Introduction to Probability

1.

(a)

$$\begin{aligned} 1 &= \int_{-1}^1 c(1-x^2) dx \\ &= c \left( x - \frac{1}{3}x^3 \right) \Big|_{-1}^1 \\ &= \frac{4}{3}c \\ \Rightarrow c &= \frac{3}{4} \end{aligned}$$

(b)

$$\begin{aligned} F(x) &= \int_{-1}^x \frac{3}{4}(1-x^2) dx \\ &= \frac{3}{4} \left( x - \frac{1}{3}x^3 \right) \Big|_{-1}^x \\ &= \frac{3}{4} \left( x - \frac{1}{3}x^3 + \frac{2}{3} \right) \\ &\quad -1 < x < 1 \end{aligned}$$

6.(b)

$$\begin{aligned} E(X) &= \int_{-1}^1 x \frac{3}{4}(1-x^2) dx \\ &= 0 \end{aligned}$$

by symmetry.

15.

(d)

$$\begin{aligned} P(X < 20) &= P\left(Z < \frac{20-10}{6}\right) \\ &\approx \Phi(1.67) \\ &\approx 0.9525 \end{aligned}$$

(e)

$$\begin{aligned}P(X > 16) &= 1 - P(X < 16) \\&= 1 - P\left(Z < \frac{16 - 10}{6}\right) \\&= 1 - \Phi(1) \\&\approx 0.1587\end{aligned}$$

20.(c)

$$\begin{aligned}P(X \leq 74.5) &= P\left(Z \leq \frac{74.5 - 100(0.65)}{\sqrt{100(0.65)(1 - 0.65)}}\right) \\&= P\left(Z \leq \frac{74.5 - 65}{4.7697}\right) \\&\approx \Phi(1.99) \\&\approx 0.9767\end{aligned}$$

21.

(a)

$$\begin{aligned}P(X > 74) &= P\left(Z > \frac{74 - 71}{\sqrt{6.25}}\right) \\&= 1 - \Phi(1.2) \\&\approx 1 - 0.8849 \\&= 0.1151\end{aligned}$$

(b)

$$\begin{aligned}P(X > 77 | X > 72) &= \frac{P(X > 77 \& X > 72)}{P(X > 72)} \\&= \frac{P(X > 77)}{P(X > 72)} \\&= \frac{P\left(Z > \frac{77 - 71}{\sqrt{6.25}}\right)}{P\left(Z > \frac{72 - 71}{\sqrt{6.25}}\right)} \\&= \frac{1 - \Phi(2.4)}{1 - \Phi(0.4)} \\&\approx \frac{1 - 0.9918}{1 - 0.6554} \\&= \frac{0.0082}{0.3446} \\&\approx 0.238\end{aligned}$$

23.

$$\begin{aligned}P(149.5 \leq X \leq 200.5) &= P(X \leq 200.5) - P(X \leq 149.5) \\&= P\left(Z \leq \frac{200.5 - 1000(1/6)}{\sqrt{1000(1/6)(5/6)}}\right) - P\left(Z \leq \frac{149.5 - 1000(1/6)}{\sqrt{1000(1/6)(5/6)}}\right) \\&\approx \Phi(2.87) - \Phi(-1.46) \\&\approx 0.9258\end{aligned}$$

25.

$$\begin{aligned}P(X \leq 10.5) &= P\left(Z \leq \frac{10.5 - 150(0.05)}{\sqrt{150(0.95)(0.05)}}\right) \\&\approx \Phi(1.12) \\&\approx 0.8686\end{aligned}$$

32.

(a)

$$\begin{aligned}P(X > 2) &= 1 - P(X < 2) \\&= 1 - \int_0^2 \frac{1}{2}e^{-x/2} dx \\&= 1 + e^{-x/2} \Big|_0^2 \\&= e^{-1}\end{aligned}$$

(b)

$$\begin{aligned}P(X \geq 10 | X \geq 9) &= \frac{P(X \geq 10 \& X \geq 9)}{P(X \geq 9)} \\&= \frac{P(X \geq 10)}{P(X \geq 9)} \\&= \frac{1 - P(X < 10)}{1 - P(X < 9)} \\&= \frac{1 - \int_0^{10} \frac{1}{2}e^{-x/2} dx}{1 - \int_0^9 \frac{1}{2}e^{-x/2} dx} \\&= \frac{e^{-5}}{e^{-4.5}} \\&= e^{-0.5}\end{aligned}$$

33.

$$\begin{aligned}P(X > 8) &= 1 - P(X < 8) \\&= 1 - \int_0^8 \frac{1}{8}e^{-x/8} dx \\&= e^{-1}\end{aligned}$$