A DISTRIBUTION-FREE TEST FOR PARALLELISM

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1. Introduction and summary. Consider the linear model

$$(1.1) \qquad Y_{ij} = \alpha_i + \beta_i X_{ij} + e_{ij}, \qquad i = 1,2; \; j = 1,2,\ldots,N$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2$ are unknown parameters and the X's are known constants. The Y's are observable while the e's are mutually independent unobservable random variables with distribution functions $P(e_{ij} \leq t) = F_i(t)$. Here we propose a method for constructing a test of

$$(1.2) \qquad H_0: \; \beta_1 = \beta_2,$$

which is distribution-free under H_0 when $(F_1, F_2) \in \mathcal{F}$ where

$$(1.3) \qquad \mathcal{F} = \{(F_1, F_2): \; F_1 \text{ and } F_2 \text{ are continuous}\},$$

thus providing an exact test of H_0 under very general assumptions.

The test consists of applying the Wilcoxon signed rank statistic [10] to $N/2$ independent estimators of $\beta_1 - \beta_2$, where each estimator is of the form $[(Y_{1s} - Y_{1s'})/(X_{1s} - X_{1s'})] - [(Y_{2t} - Y_{2t'})/(X_{2t} - X_{2t'})]$. The procedure is similar in character to a recent proposal of Olshen [6] for testing linearity against convexity. It has the disadvantage of depending on irrelevant randomizations and, at first glance, appears to be extremely wasteful of the information in the data. However, for the equally spaced model

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(3.2) of section 3, where on line i we take $(N/2k)$ observations at each of the $2k$ points $C_i + 2j$, $j = 0,\ldots,2k - 1$, the Pitman efficiency of the signed rank test with respect to the normal theory t-test, for the case $F_1 = F_2 = F$ (say) and F normal, is .955 for $k = 1$ and $\to .716$ as $k \to \infty$. The corresponding values for F uniform and F exponential are, respectively, (.919, .689) and (1.172, .879). For the same model with F_1 uniform and F_2 exponential, these efficiencies are (1.437, 1.077).

Potthoff [7] has developed a "nonparametric" but non-distribution-free test of H_0 which may be considered a competitor of the signed rank test proposed here. His procedure is not restricted to designs with equal numbers of observations on each line but its applicability is limited by the requirement that no two X_1's be equal and no two X_2's be equal. In section 4 we present some Monte Carlo calculations which find Potthoff's test lagging significantly behind the signed rank test and the t-test with respect to power.

2. The signed rank procedure. Assume $N = 2n$ and for line i form n groups, each containing two X's by pairing the X_{ij}'s. For each group compute a slope estimator of β_i of the form $(Y - Y')/(X - X')$ where Y, Y' are the observations corresponding to the paired X, X' points. The pairings cannot depend on the observed Y's but only on the $\{X_{ij}\}$ configurations. Different Y's are used for each estimator and thus this leads to n mutually independent slope estimators for line i, which we call $u_{i1},\ldots,u_{in}$. Of course the line 1 slope estimators are also independent of the line 2 slope estimators. Denote the distance between the paired X's used in the determination of u_{ij} by d_{ij}. The pairings must be such that all the d's are

nonzero. Now pair the u_1's with the u_2's and for each of the n pairs compute a difference of the form $w = u_1 - u_2$. Call these differences $w_1,\ldots,w_n$. Again, this pairing can depend only on the $\{d_{1j}\}$ and $\{d_{2j'}\}$ constants, and not on the values of the u's. Random pairing falls under this framework and is suggested for the equally spaced design discussed in section 3.

The test statistic is the Wilcoxon signed rank statistic applied to the w's, which (using a representation due to Tukey [9]) can be written in the form,

$$(2.1) \qquad W_n = \sum_{i\leq j}^{n} \phi(w_i + w_j),$$

where $\phi(a) = 1$ if $a < 0$, 0 otherwise. Significantly large values of W_n will be considered indicative of situations where $\beta_2 > \beta_1$ with the opposite interpretation for small values. While the non-null distribution of W_n will depend on the pairing schemes used, the d's, and (F_1,F_2), under H_0 we have

Theorem 1. When H_0 is true and $(F_1,F_2) \in \mathcal{F}$, W_n has the usual null distribution of Wilcoxon's signed rank statistic, that is W_n has the same distribution as $\sum_{i=1}^{n} i\delta_i$ where the $\{\delta_i\}$ are independent and identically distributed (iid) with $P(\delta_i = 1) = P(\delta_i = 0) = 1/2$.

Proof. This is an immediate consequence of signed rank test theory. We need only note that (i) although the w's are, in general, not identically distributed, they are independent, (ii) when $(F_1,F_2) \in \mathcal{F}$, each w has a distribution which is continuous and symmetric about $\beta_1 - \beta_2$, and (iii) $\beta_1 - \beta_2$ is 0 under H_0.

It follows that $[W_n - (n(n+1)/4)]/[n(n+1)(2n+1)/24]^{1/2}$ is asymptotically $N(0,1)$ under H_0. An application of a theorem of Sen [8] gives the asymptotic normality of $W_n' = \sum_{i<j}^{n} \phi(w_i + w_j)$ for a sequence of alternatives approaching the null hypothesis. (It is well known that W_n and W_n' are asymptotically equivalent (cf. [4]).) Define

(2.2) $\quad H_n: \quad \theta_n = \beta_{2(n)} - \beta_{1(n)} = \theta/n^{1/2},$

where θ is real, and make the assumptions A_1: F_i is absolutely continuous with square-integrable continuous density f_i, $i = 1,2$, and A_2: $\inf_{n}\inf_{j}\{d_{ij}\} > 0$, $i = 1,2$. Let g_i denote the density of w_i and define $\bar{g}_n(t) = \sum_{i=1}^{n} g_i(t)/n$.

Theorem 2. Under A_1, A_2, and H_n, $3^{1/2}\{n^{1/2}[(W_n/\binom{n}{2}) - \frac{1}{2}] - \gamma_n\}$ is asymptotically $N(0,1)$ uniformly in F_1, F_2, and $\{d_{ij}\}$, where $\gamma_n = 2\theta(\int_{-\infty}^{\infty} \bar{g}_n^{\,2}(t)dt)$.

Proof. Each g_i is symmetric about θ_n, A_1 and A_2 imply g_i is continuous and $\sup_i \int_{-\infty}^{\infty} g_i^{\,2}(t)dt < \infty$, and the theorem then follows by a direct application of Theorem 2.1 of Sen [8].

3. Pitman efficiencies for an equally spaced design. An exact normal theory of H_0, specialized to the case of equal numbers of observations on each line, is based on

(3.1) $\quad t_n = (b_2 - b_1)/sZ,$

where $b_i = \sum_{j=1}^{N} (Y_{ij} - Y_{i.})(X_{ij} - X_{i.}) / \sum_{j=1}^{N} (X_{ij} - X_{i.})^2$,

$s^2 = [(N-2)s_1^2 + (N-2)s_2^2]/(2N-4)$, $s_i^2 = \sum_{j=1}^{N} (Y_{ij} - a_i - b_i X_{ij})^2/(N-2)$,

$a_i = Y_{i.} - b_i X_{i.}$, and $Z^2 = \sum_{i=1}^{2} [1/\sum_{j=1}^{N} (X_{ij} - X_{i.})^2]$. We use the subscript n in (3.1) but note that here t_n is computed for $N = 2n$ observations on each line. When F_1 and F_2 are normal with the same variance and H_0 is true, t_n has the Student-t distribution based on $2N-4$ degrees of freedom.

To compare the t_n and W_n tests we consider the equally spaced design where on line i we have a total of $N = 2kn'$ observations with n' observations at each of the 2k points

$$(3.2) \qquad C_i + 2c_i j, \qquad j = 0,\ldots,2k-1$$

where C_i, c_i are arbitrary constants with $c_i > 0$, $i = 1,2$. For this design we form W as follows. Take the k intervals

$$(3.3) \qquad [C_i + 2c_i j,\ C_i + 2c_i(j + k)], \qquad j = 0,\ldots,k-1$$

and for each interval obtain n' independent slope estimators of β_i by pairing at random a Y at the lower endpoint with a Y' at the upper endpoint and computing $u = (Y' - Y)/2kc_i$. This yields the $n = (N/2) = kn'$ independent slope estimators $u_{i1},\ldots,u_{in}$ for each line. Then randomly pair the u_1's with the u_2's to form the w differences of section 2.

Theorem 3. For the design (3.2) and the grouping scheme (3.3), the Pitman efficiency $E_{W,t}$ of W_n with respect to t_n for the alternatives H_n (2.2) is

$$(3.4) \qquad E_{W,t} = \{72k^2(c^2\sigma_1^2 + \sigma_2^2)[\int_{-\infty}^{\infty} h_1(t)h_2(ct)dt]^2\}/(4k^2 - 1),$$

where $\sigma_i^2 = \mathrm{Var}(F_i)$, $c = (c_2/c_1)$, and $h_i(t) = (d/dt)H_i(t)$ where H_i is the distribution function of $T_{i1} + T_{i2} - T_{i3} - T_{i4}$ when $T_{i1}, T_{i2}, T_{i3}, T_{i4}$ are iid according to F_i, $i = 1,2$.

Proof. From [5] we find

$$(3.5)\qquad E_{W,t} = \lim_n\{[(d/d\theta)E_\theta(W_n')|_{\theta=0}][(d/d\theta)E_\theta(t_n)|_{\theta=0}]^{-1}\}^2[\mathrm{Var}_0(t_n)/\mathrm{Var}_0(W_n')],$$

where the subscript θ indicates the expectation is computed under $\beta_2 - \beta_1 = \theta$. Now, $E_\theta(W_n') = \sum_{i<j}^{n} P_\theta(w_i + w_j < 0)$, and in this case the w's are identically distributed so we obtain,

$$E_\theta(W_n') = (n(n-1)/2)P\left(-2\theta + \frac{e_{11}'-e_{12}'}{2kc_1} + \frac{e_{13}'-e_{14}'}{2kc_1} - \frac{(e_{21}'-e_{22}')}{2kc_2} - \frac{(e_{23}'-e_{24}')}{2kc_2} < 0\right)$$

(3.6)

$$= (kn'(kn'-1)/2)\left\{\int_{-\infty}^{\infty} H_1(t+4\theta kc_1)dH_2(c_2c_1^{-1}t)\right\},$$

where e_{i1}', e_{i2}', e_{i3}', e_{i4}' are iid according to F_i. Hence we have

$$(3.7)\qquad (d/d\theta)E_\theta(W_n')|_{\theta=0} = (kn'(kn'-1)/2)\lim_{\delta\to 0}\int_{-\infty}^{\infty}\{[H_1(t+4\delta kc_1)-H_1(t)]/\delta\}dH_2(ct).$$

Square-integrability of h_1 and h_2 allows the evaluation of (3.7) by differentiation under the integral in (3.6). The justification follows in the same manner as Lemma 3.4 of Mehra and Sarangi [4]. We have

$$(3.8)\qquad \left|\int_{\infty}^{\infty}\{[H_1(t+4\delta kc_1) - H_1(t)]/\delta\}dH_2(ct) - 4kcc_1\int_{-\infty}^{\infty} h_1(t)h_2(ct)dt\right|^2$$

$$(3.9)\qquad \leq 16k^2(cc_1)^2\left[\int_{-\infty}^{\infty} h_2^2(ct)dt\right]\int_{-\infty}^{\infty}\{([H_1(t+4\delta kc_1) - H_1(t)]/4\delta kc_1) - h_1(t)\}^2dt,$$

the inequality being a consequence of Schwarz's inequality. The first integral in (3.9) is finite since h_2 is square-integrable, the second $\to 0$ as $\delta \to 0$ by Lemma 4.3 of Hájek [1] since h_1 is square-integrable. Thus from (3.7), (3.8), (3.9) we obtain

$$(3.10) \qquad (d/d\theta)E_\theta(W_n')\big|_{\theta=0} = 2k^2n'(kn'-1)cc_1 \int_{-\infty}^{\infty} h_1(t)h_2(ct)dt.$$

Also, $\mathrm{Var}_0(W_n) = kn'(kn'+1)(2kn'+1)/24$, $(d/d\theta)E_\theta(t_n)\big|_{\theta=0} \sim [(\sigma_1^2 + \sigma_2^2)Z^2/2]^{-\frac{1}{2}}$, where $Z^2 = [3(c_1^2 + c_2^2)]/[2n'kc_1^2c_2^2(4k^2 - 1)]$ and $\mathrm{Var}_0(t_n) \sim [(\sigma_1/c_1)^2 + (\sigma_2/c_2)^2] \div \{[c_1^{-2} + c_2^{-2}][(\sigma_1^2 + \sigma_2^2)/2]\}$. Equation (3.4) now follows from (3.5).

When $c = 1$ and $F_1 = F_2$, $E_{W,t}$ reduces to $3k^2\{12\tau_1^2[\int_{-\infty}^{\infty} h_1^2(t)dt]^2\}/(4k^2-1)$ where $\tau_1^2 = 4\sigma_1^2 = \int_{-\infty}^{\infty} t^2h_1(t)dt$. From Hodges and Lehmann [2] it follows that $E_{W,t}$ can be infinite and a lower bound in the case $c = 1$, $F_1 = F_2$, is given by

$$(3.11) \qquad E_{W,t} > [3k^2(108/125)]/[4k^2-1].$$

Although $12\tau_1^2[\int_{-\infty}^{\infty} h_1^2(t)dt]^2 = (108/125)$ when $h_1(t) = (3/20(5)^{1/2})(5-t^2)$ for $-5^{1/2} \leq t \leq 5^{1/2}$ and 0 otherwise, it is easily seen that this density cannot be the density of a random variable of the form $S_1 - S_2$ where S_1, S_2 are independent and identically distributed, and hence the inequality in (3.11) is strict. The lower bound given by (3.11) is a decreasing function of k which equals .864 when $k = 1$ and tends to .648 as $k \to \infty$. Table 1 contains values of (3.4) for various (F_1, F_2) pairs. In Table 1 and Table 2 (section 4) the letters N, U, E denote, respectively, the distributions with density functions $f(x) = (2\pi)^{-1/2} \exp(-x^2/2)$ for $-\infty < x < \infty$; $f(x) = 1$ for $-1/2 \leq x \leq 1/2$ and 0 otherwise; $f(x) = 2^{-1} \exp(-x/2)$ for $x > 0$ and 0 otherwise. Note that for fixed (F_1, F_2) and c, the efficiencies are decreasing in k, corresponding to the increased loss of information due to grouping. When $(F_1, F_2) = (N, N)$, the efficiencies are independent of c. The $k = c = \infty$ entries correspond to $\lim_{k\to\infty} \lim_{c\to\infty} E_{W,t}$.

TABLE I

$E_{W,t}$ values for the equally spaced design (3.2)

c \ k	$(F_1,F_2) = (E,E)$ 1	2	4	10	∞	$(F_1,F_2) = (U,U)$ 1	2	4	10	∞
1	1.172	.938	.893	.881	.879	.919	.735	.700	.691	.689
2	1.245	.996	.948	.936	.934	.905	.724	.690	.681	.679
4	1.373	1.098	1.046	1.032	1.029	.893	.714	.680	.671	.670
10	1.468	1.174	1.118	1.103	1.101	.889	.711	.677	.669	.667
∞	1.500	1.200	1.143	1.128	1.125	.889	.711	.677	.688	.667

c \ k	$(F_1,F_2) = (U,E)$ 1	2	4	10	∞	$(F_1,F_2) = (N,N)$ 1	2	4	10	∞
1	1.437	1.149	1.095	1.080	1.077	.955	.764	.728	.718	.716
2	1.325	1.060	1.010	.997	.994	.955	.764	.728	.718	.716
4	1.141	.913	.880	.858	.856	.955	.764	.728	.718	.716
10	.947	.758	.721	.712	.710	.955	.764	.728	.718	.716
∞	.889	.711	.677	.668	.667	.955	.764	.728	.718	.716

4. Potthoff's test. Potthoff [7] has proposed a test of H_0 based on

$$(4.1)\qquad P = \left[\binom{N_1}{2}\binom{N_2}{2}\right]^{-1} \sum_{s<s'} \sum_{t<t'} \phi\left([(Y_{1s}-Y_{1s'})/(X_{1s}-X_{1s'})] - [(Y_{2t}-Y_{2t'})/(X_{2t}-X_{2t'})]\right),$$

where there are N_i Y's on line i and it is assumed that no X_1's are equal and that no X_2's are equal. He shows $E_0(P) = 1/2$, $Var_0(P)$ depends on the X's and (F_1,F_2), and $\sup_{(F_1,F_2)\in\mathcal{F}} Var_0(P) = (2M+5)/18M(M-1)$ where $M = Min[N_1,N_2]$. Potthoff establishes the asymptotic normality of P under H_0 with a mild restriction on the X_{ij}'s (Assumption 5E of [7]). His α-level test is "conservative"; the one-sided test against $\beta_2 > \beta_1$ would have as critical region $(P - \frac{1}{2})[(2M+5)/18M(M-1)]^{-1/2} > z^{1-\alpha}$ where $z^{1-\alpha}$ is the $1 - \alpha$ percentile point of a $N(0,1)$ distribution. This procedure is neither distribution-free nor asymptotically distribution-free.

Monte Carlo sampling was used to investigate the power of the P test. For the design (3.2) with $k = 10$, $n' = 1$, $c_1 = c_2 = 1$, Table 2 gives the relative frequencies with which the one-sided t, W, and P tests rejected H_0 in favor of $\beta_2 > \beta_1$, for various (F_1,F_2) pairs, $\alpha = .0098$, $\alpha = .0527$, and several values of $\Delta = (\beta_2 - \beta_1)/\{[(\sigma_1^2 + \sigma_2^2)/2]\sum_{i=1}^{2}[1/\sum_{j=1}^{2kn'}(X_{ij} - X_{i\cdot})^2]\}^{1/2}$. For each α, Δ, and (F_1,F_2) combination, the power estimates are based on 500 samples with each sample consisting of 20 random values from F_1 and 20 from F_2. For each α, the four Δ values were chosen so that the exact powers of the t-test for $(F_1,F_2) = (N,N)$ would be α, .3, .7, and .9. This provides a check on the Monte Carlo values as does the fact that the exact power of the W test when $\Delta = 0$ is α for each (F_1,F_2) pair. Furthermore, the Monte Carlo power values for W when $(F_1,F_2) = (N,N)$ are consistent with the exact power values of the signed rank test against normal shift alternatives given by Klotz in Table 1 of [3]. (For the design (3.2) with $n' = 1$, $c_1 = c_2 = 1$, Klotz's N corresponds to our k and his μ corresponds to our $\Delta[3k/(4k^2-1)]^{1/2}$.)

TABLE 2

Estimated power for the equally spaced design
$k = 10,\ n' = 1,\ c_1 = c_2 = 1$

$\alpha = .0098$

Δ	0			1.90			3.01			3.79		
Test	t	W	P	t	W	P	t	W	P	t	W	P
(F_1,F_2)												
(N,N)	.012	.024	.000	.274	.168	.054	.694	.440	.298	.914	.652	.598
(U,U)	.008	.008	.002	.262	.124	.058	.732	.408	.292	.914	.678	.544
(E,E)	.014	.004	.000	.316	.186	.010	.734	.468	.204	.896	.670	.580
(U,E)	.014	.016	.000	.414	.254	.032	.754	.484	.242	.922	.694	.592

$\alpha = .0527$

Δ	0			1.14			2.21			2.98		
Test	t	W	P	t	W	P	t	W	P	t	W	P
(F_1,F_2)												
(N,N)	.038	.040	.016	.302	.250	.130	.728	.560	.470	.881	.728	.706
(U,U)	.050	.064	.006	.260	.214	.040	.698	.532	.286	.910	.724	.652
(E,E)	.064	.052	.000	.306	.276	.046	.694	.584	.288	.908	.808	.660
(U,E)	.052	.044	.000	.354	.308	.036	.750	.638	.306	.902	.812	.688

The indication from Table 2 is that the Potthoff test is too conservative, the $\Delta = 0$ entries being far below the nominal α value. The estimated powers show that replacing the unknown null variance by $\sup_{(F_1,F_2)\in\mathcal{F}} \mathrm{Var}_0(P)$ is unsatisfactory. The fact that the estimated significance levels of the t-test are quite close to the nominal levels is not surprising. The t-test is exact when F_1 and F_2 are normal with the same variance. However, when $\sigma_1^2 < \infty$ and $\sigma_2^2 < \infty$, t (3.1) is asymptotically normal with asymptotic mean 0 under H_0, and asymptotic variance

$$\left\{\sum_{i=1}^{2}\left[\sigma_i^2/\sum_{j=1}^{N}(X_{ij} - X_{i.})^2\right]\right\}\Big/\left\{\left[(\sigma_1^2 + \sigma_2^2)/2\right]\sum_{i=1}^{2}\left[1/\sum_{j=1}^{N}(X_{ij} - X_{i.})^2\right]\right\}.$$

This asymptotic variance is 1 when $\sum_{j=1}^{N}(X_{1j} - X_{1.})^2 = \sum_{j=1}^{N}(X_{2j} - X_{2.})^2$ which is the case for the design explored in Table 2. Hence the t-test is asymptotically exact for this design and the four (F_1,F_2) pairs considered.

5. Conclusion. The signed rank procedure suggested here should be of interest to those who feel the dependence on randomization may be a fair price to pay for the exact test. The Pitman efficiencies provide confidence in the use of W for equally spaced or nearly equally spaced designs. We emphasize that the design (3.2) includes two cases frequently encountered in practice, namely k = 1 (the two point design) and n´ = 1 (the equally spaced no replications case). While we have not specified the groupings to be used in the formation of W for unequally spaced designs, the recommendation is to approximate the grouping scheme (3.3) used in the equally spaced case. Finally, although we have made no progress for the problem of unequal numbers of observations on each line, the Monte Carlo values of section 4 do imply that Potthoff's test does not provide a satisfactory solution.

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13. Abstract:

Consider two regression lines with slopes β_1, β_2. We propose a method for constructing a test of H_0: $\beta_1 = \beta_2$ which is distribution-free under H_0 when the error distributions are continuous. The test consists of applying the Wilcoxon signed rank statistic to certain independent estimators of $\beta_1 - \beta_2$. Power comparisons are presented for the signed rank test, a normal theory t-test, and a test proposed by Potthoff.

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