Nonparametric Estimation Under Shape Constraints: Monotone, Convex, and Beyond

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by

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Abstract: In this talk I will consider nonparametric estimation of an unknown density function g under shape constraints from a mixture model perspective. Let k be a non-negative integer and let F be a distribution on $(0, \infty)$. Then

$$g(x) = \int_0^\infty \frac{k+1}{y^{k+1}}(y-x)^k 1_{[0,y]}(x) dF(y)$$

is monotone (decreasing) when $k = 0$, g is convex and decreasing when $k = 1$, and higher values of k correspond to densities which are k times differentiable with derivatives of alternating sign. I will discuss what is known concerning estimation of g and the mixing distribution F when $k = 1$ and $k = 2$, and then discuss open problems connected with the cases $k \geq 3$ and $k = \infty$.