

# Nonparametric Estimation Under Shape Constraints: Monotone, Convex, and Beyond

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by

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**Abstract:** In this talk I will consider nonparametric estimation of an unknown density function  $g$  under shape constraints from a mixture model perspective. Let  $k$  be a non-negative integer and let  $F$  be a distribution on  $(0, \infty)$ . Then

$$g(x) = \int_0^\infty \frac{k+1}{y^{k+1}} (y-x)^k 1_{[0,y]}(x) dF(y)$$

is monotone (decreasing) when  $k = 0$ ,  $g$  is convex and decreasing when  $k = 1$ , and higher values of  $k$  correspond to densities which are  $k$  times differentiable with derivatives of alternating sign. I will discuss what is known concerning estimation of  $g$  and the mixing distribution  $F$  when  $k = 1$  and  $k = 2$ , and then discuss open problems connected with the cases  $k \geq 3$  and  $k = \infty$ .