ERRORS-IN-VARIABLES : THE NONLINEAR MODEL WITH AN ERROR IN THE EQUATION

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Abstract

In a lot of situations : medicine, social sciences... variables of interest X can not be observed directly. Applications in which the X-variable is measured with error is perhaps more common than those in which X is measured precisely. For example variables such as blood pressure, pulse rate, age of the fetus, dose of caffeine absorbed every day, soil nitrogen concentration... are measured with non negligible error, so one observes $X^* = X + U$ where U is an additive error. However informations on U are available by resampling data, conducting others experiments, using others variables...

The classical errors-in-variables (EV) model is

$$y = \beta X,$$

where one observes X^* and y^* which are the true variables plus the additive errors U and v

$$X^* = X + U \quad \text{and} \quad y^* = y + v,$$

with assumptions on the errors.

The true values X and y may not be perfectly related (if for example factors other than X are responsible for variation in y). Thus Fuller (1987) proposed the model with an error in the equation

$$y = X\beta + q \quad q \sim \mathcal{N}(O, \sigma_{qq}),$$

where one observes $(y^*, X^*) = (y, X) + (v, U)$. The vector of measurement errors $A = (v, U)$ has a Gaussian distribution with mean zero and a covariance matrix Σ_{AA} and is independent of (q, X). When the relation between X and y is not linear Fuller (1987, p261.) proposed the nonlinear model with and error in the equation

$$y = g(X, \beta) + q \quad q \sim \mathcal{N}(O, \sigma_{qq}),$$

where one observes $(y^*, X^*) = (y, X) + (v, U)$. $A = (v, U)$ is the vector of measurement errors. (q, A) is independent of X with

$$(q, A) \sim \mathcal{N}(0, blocdiag(\sigma_{qq}, \Sigma_{AA})).$$

He assumed also that Σ_{AA} is nonsingular and known and that the form of $g(.,.)$ is known.

The aim of the presentation is to estimate the function g nonparametically. Our estimator is based on deconvoluted kernels. Asymptotic results will be presented and some simulations will show that our estimator is tractable and performs relatively well in practice.

References

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